An Investigation of Residual Stresses in Tempered Glass Plates of Aircraft Windshields

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A scattered-light photoelastic method that determines nondestructively the distribution of residual stresses across the thickness in tempered glass plates of aircraft windshields is presented. This paper begins with an introduction to scattered-light photoelasticity and a discussion of the state of stress in tempered glass plates. Then, the theory of the new method is developed. Apparatus and techniques are described. Experiments were made on an aircraft windshield sample having a three-layer (glass-vinyl-glass, thermally tempered) sandwich structure. Photographs of photoelastic fringe pattern in scattered light were taken and the distribution of residual stresses across the thickness of the glass layer was obtained. The accuracy of the method is established from the fact that a good agreement exists between results of different experiments and that the condition of equilibrium is satisfied.

Nomenclature

A	= amplitude of vibration
C_1,C_2	= transverse and direct stress-optic coefficients, respectively
I	= intensity of light
K	= constant
n	= relative retardation, or fringe order
p,q,r	= principal stresses
p',q'	= secondary principal stresses
s	= length of light path
t	= time
U,V	= vibrational components along p' and q' directions
X,Y,Z	= coordinate axes
α	= angle between the direction of polarization of incident light and the U direction, Fig. 1
β	= angle between the plane of incidence and the XZ plane, Fig. 2
γ	= angle between the U direction and the direction of observation
θ	= angle of incidence
λ	= wavelength of light
μ_o	= refractive index of the unstressed material
μ_p,μ_q,μ_r	= refractive indices along p, q, and r directions, respectively
ϕ	= phase angle
ω	= angular velocity of vibration

Introduction

GLASS is a brittle material. It always breaks in tension.[1] The theoretical strength of glass based on interatomic forces is 3–4 $\times$ 10^6 psi. A strength of about 2 $\times$ 10^6 psi has been observed from fused silica fibers in liquid nitrogen.[2] Under normal ambient conditions, glass, in bulk form, possesses tensile strength between 5000 and 30,000 psi. The low strength has been attributed to surface flaws which act as stress concentrators and initiate fracture. On the other hand, glass is so strong in compression that no definite compressive strength has yet been established, although a value of several hundred thousand psi has been observed.[3]

Under service conditions, one surface of the windshield is under compression and the other surface is under tension. The tempering procedure introduces high compressive stress in both surfaces which must be overcome before a tensile stress of sufficient magnitude can affect failure. The strength of windshields is thus increased many times.

A survey of the literature shows that numerous attempts have been made in the past to determine nondestructively the strength of tempered glass plates. However, little work was done in employing photoelasticity for this purpose. In 1956, Acloque and Guillemet[4] reported an oblique incident method in transmitted light photoelasticity. They determined the compressive stress at the surface of thermally tempered glass plates based on the assumption of parabolic stress distribution across the thickness of the plate. In 1966, Bateson, Hunt, Dalby, and Sinha[5] determined some principal stress differences at the middle plane of glass plates by means of scattered-light photoelasticity. In 1967, Cheng[6] developed a scattered-light photoelastic method which determines nondestructively the distribution of residual stresses across the thickness of tempered glass plates. This information could be used to estimate the strength of the plates.

Scattered-Light Photoelasticity

In 1816 David Brewster discovered that glass becomes doubly refracting under stress or strain. Whether this phenomenon depends upon stress or strain is not important provided that the material remains within the proportional limit. He showed that this property is identical with those found in crystals.

It has long been known[7] that, upon entering a doubly refracting material, a ray of light is resolved into two components which polarize along different directions, travel with different velocities according to the principal refractive indices of the material, and emerge with a relative retardation between these components. The relative retardation is the birefringence, which manifests itself as colored fringes in a field of white light. If the light is monochromatic, they appear as bright and dark fringes.

In crystalline materials the permanent birefringence may be described by the index ellipsoid whose principal axes are proportional to the principal refractive indices. In materials exhibiting temporary birefringence due to stresses within the proportional limit, the principal stresses p,q,r are in the directions of the principal axes of the index ellipsoid and are related to the refractive indices as follows[8]:

$$\mu_p - \mu_o = C_1p + C_2(q + r) \tag{1a}$$

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$$\mu_q - \mu_o = C_1 q + C_2(r + p) \tag{1b}$$

$$\mu_r - \mu_o = C_1 r + C_2(p + q) \tag{1c}$$

Subtracting pairs of Eqs. (1), we obtain

$$\mu_p - \mu_q = C'(p - q) \tag{2a}$$

$$\mu_q - \mu_r = C'(q - r) \tag{2b}$$

$$\mu_r - \mu_p = C'(r - p) \tag{2c}$$

where $C' = C_1 - C_2$. The relative retardation n resulting from a ray of monochromatic light passing through a length of light path s is

$$n = Cs(p - q) \tag{3a}$$

$$n = Cs(q - r) \tag{3b}$$

or

$$n = Cs(r - p) \tag{3c}$$

where $C = C'/\lambda$. These relations are for rays parallel to the directions of r,p,q, respectively. For rays of arbitrary direction the relation is given by[9]

$$n = Cs(p' - q') \tag{4}$$

where p' and q' are the secondary principal stresses for the given ray. Equation (4) gives the relation between n and $(p' - q')$ for a general ray in transmitted light. There is one more relation which gives the directions of p' and q'. The state of stress is then determined from these relations in combination with the equation of equilibrium.

Scattered-Light Photoelasticity

It is known that, when light travels through a cloudy medium, one can see the illuminated particles indicating the light path. Light is said to be scattered in all directions. A basic characteristic of scattered light is that it is always linearly polarized.

In 1939, Weller[10] showed that it is possible to obtain sufficient photoelastic data, i.e., the directions of and the difference between the secondary principal stresses, for determining the state of stress from observations of scattered light along directions normal to the direction of propagation of the light ray. This method is nondestructive. The stress-optic relation is given by

$$dn/ds = C(p' - q') \tag{5}$$

where dn/ds is the fringe gradient along the light path.

In this method, the birefringence is the average optical effect across the diameter of the light pencil. Hence, the smaller the diameter is, the higher the accuracy will be.

Formation of Photoelastic Fringe Pattern

The intensity of scattered light depends, among other things, on the direction of observation. In the following discussion we shall limit ourselves to cases where the directions of observation are normal to the direction of propagation of the light ray, i.e., the observations are in transverse planes. In this case, the intensity is proportional to the square of the apparent amplitude, which is defined as the amplitude of the vibrational component normal to the direction of observation.

If the material through which a ray of linearly polarized light $A\cos\omega t$ travels is stressed, the ray is resolved at the point of entrance into two components, $A\cos\alpha\cos\omega t$ and $A\sin\alpha\cos\omega t$, along the directions of p' and q' in the transverse plane (Fig. 1). These components travel with different velocities. The velocity difference is proportional to $(p' - q')$ and introduces a phase difference ϕ. At a point along the

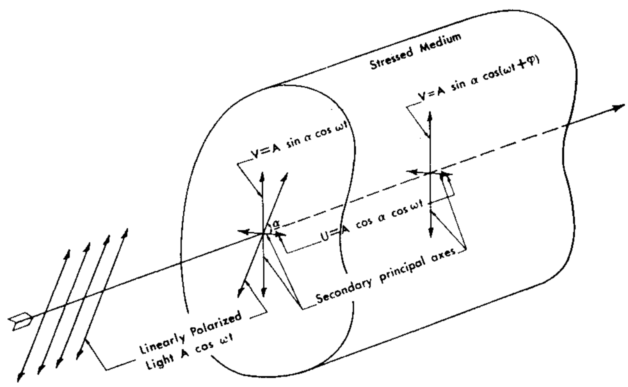


Fig. 1 Transmission of linearly polarized light through a stressed medium.

light ray, these components have the following form:

$$U = A\cos\alpha\cos\omega t \tag{6a}$$

$$V = A\sin\alpha\cos(\omega t + \phi) \tag{6b}$$

This expression shows that the ray, in general, is elliptically polarized.

It can be shown (see Appendix) that the intensity of scattered light for an arbitrary direction of observation in the transverse plane is given by

$$I = KA^2(\cos^2\alpha\sin^2\gamma + \sin^2\alpha\cos^2\gamma - \tfrac{1}{2}\sin 2\alpha\sin 2\gamma\cos\phi) \tag{7}$$

This intensity reaches a maximum or a minimum when the direction of observation is along the axes of the light ellipse. For a given direction of observation, the intensity depends on the value of ϕ. In general, ϕ varies from point to point. Hence, a sequence of bright and dark region appears to form a photoelastic fringe pattern in scattered light.

Differentiating Eq. (7) with respect to γ, we obtain

$$dI/d\gamma = KA^2(\cos 2\alpha\sin 2\gamma - \sin 2\alpha\cos 2\gamma\cos\phi) \tag{8}$$

This expression shows that, for $\alpha = 45°$, the pattern attains the best contrast when γ is also equal to 45°, since maximum reinforcement or interference of light occurs in this case.

Stresses in Tempered Glass Plates

Thermal tempering consists of heating the glass to a temperature near the softening point for stresses in it to relax and subsequent rapid cooling. The transient thermal stresses and stress relaxation while the glass is still hot produce permanent stresses which serve to strengthen the glass by compressively prestressing its surfaces. The stress distribution across the thickness of the plate is approximately parabolic and is in a state of equilibrium. The maximum compressive stress occurs at the surface. An isotropic state of stress exists in planes parallel to the surface, except in regions very near the edge. Hence, no birefringence could be observed in transmitted light photoelasticity at normal incidence.

In chemically tempered glass, the parabolic distribution of permanent stresses across the thickness is not necessarily true.

New Scattered-Light Method

Let the directions of p,q,r coincide with the coordinate axes (Fig. 2). Further, let a ray of properly polarized light propagate along direction AO having an angle of incidence θ. The plane of incidence AOZ is at an angle β with the XZ plane. The secondary principal stress difference for this ray at point O is given by

$$p' - q' = p\sin^2\beta + q\cos^2\beta - [(p\cos^2\beta + q\sin^2\beta)\cos^2\theta + r\sin^2\theta] \tag{9}$$

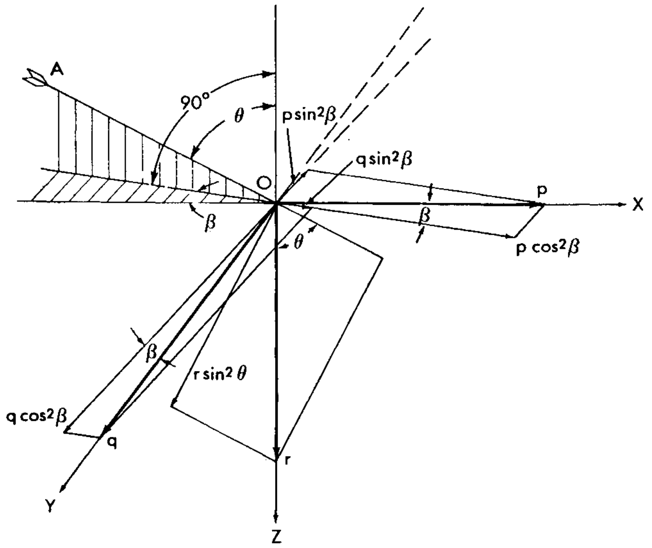


Fig. 2 Stress components.

Consider the XY plane as the surface of tempered glass plate. Because of isotropy, Eq. (9) reduces to

$$p' - q' = (q - r)\sin^2\theta \tag{10}$$

Experimental results shows that, for a thin plate, the third principal stress r is negligible. Then

$$p' - q' = q \sin^2\theta \tag{11}$$

and Eq. (5) becomes

$$dn/ds = Cq \sin^2\theta \tag{12}$$

Hence, the fringe gradient gives a measurement of the principal stresses p and q.

Specimen

The specimen was a sample of aircraft windshields. It was a three-layer (glass-vinyl-glass) sandwich structure. The outer glass layers were thermally tempered and had a thickness of $\frac{1}{2}$ and $\frac{1}{8}$ in., respectively. The central vinyl layer had a thickness of 0.3 in. Experiments were performed on the $\frac{1}{2}$-in. glass layer.

Apparatus and Techniques

Figure 3 shows a sketch of the apparatus. A helium-neon continuous-wave gas laser (Spectra-Physics 130) was employed as the light source. It emitted a linearly polarized, collimated (0.7-mrad divergence) and monochromatic (6328Å) light pencil of 1.4-mm diameter. In order to improve the accuracy, a lens having a focal length of 225 mm was employed to reduce the diameter of the light pencil with a negligible

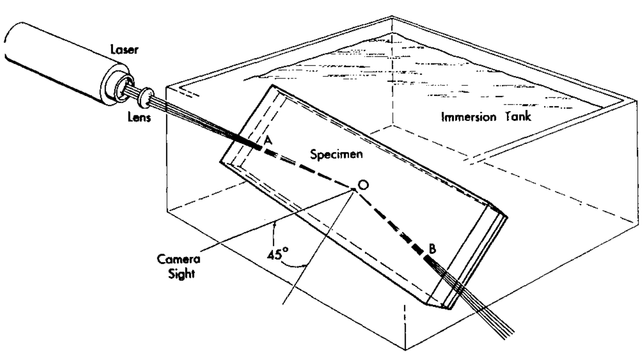


Fig. 3 Sketch showing apparatus.

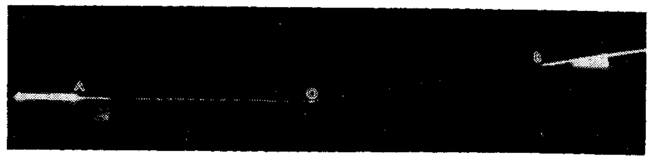


Fig. 4 Photograph of photoelastic fringe pattern in scattered light, $\theta = 84°$.

effect on the collimating characteristic of the light. Using this lens, the light can be focused to a spot having a diameter of $225 \times 0.7 \times 10^{-3} = 0.16$ mm, approximately. The linearly polarized light, having its direction of vibration in a vertical plane, propagated in a horizontal plane, entered the immersion tank normally through a window and focused at a point O on the glass-vinyl interface. Immersion fluid having the same refractive index as the glass plate was used to eliminate refraction and reflection at point A, where the light entered the glass plate. The specimen was oriented such that the normal of the plane of incidence made a vertical angle of 45° with the camera sight in a horizontal plane. Photographs of photoelastic fringe pattern were taken normal to the light path AO. The conditions for attaining the best contrast were thus satisfied. The angle of incidence θ had a suitable value. Kodak Tri-X film and an exposure time of 15 min at f8 were used.

Experimental Results

Figure 4 shows photograph of photoelastic fringe pattern in scattered light with $\theta = 84°$. Sharp fringes can be clearly seen along AO except at the vicinity of the entrance point A, where the fringes were washed out by the high intensity of light scattered in the fluid. However, this portion can be easily extrapolated since it is antisymmetrical with respect to the middle plane. The light reflected at point O, followed path OB, and left the glass plate at point B. Fringes along OB are less clear than those along OA because of the loss of intensity upon reflection.

Figure 5 gives a curve of fringe order vs light path. The fringe order was read from a 7× print. The fringe gradient was calculated and is shown in dimensionless form in Fig. 6. Since the plate thickness is equal to $AO/\sec\theta$ and the stress is equal to $(dn/ds)/(C \sin^2\theta)$, the curve in Fig. 6 also represents the distribution of p and q across the thickness of the plate.

Discussion

1) Since an isotropic state of stress exists in planes parallel to the surface of the plate and the stress is in equilibrium across the thickness of the plate, it follows that

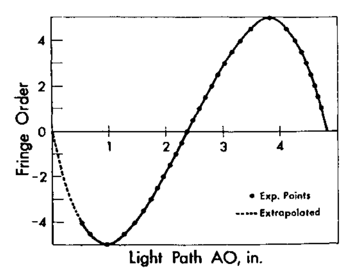
$$\int q ds = 0 \tag{13}$$

An evaluation of this expression from Fig. 6 shows an excess of 2 units out of a total of 97 units, which is an error of 2% and is within the experimental limit.

2) If the third principal stress r is not equal to zero, the curve in Fig. 6 would represent the distribution of $(q - r)$, and

$$\int (q - r)ds = \int q ds - \int r ds = -\int r ds \tag{14}$$

Fig. 5 Curve of fringe order vs light path.



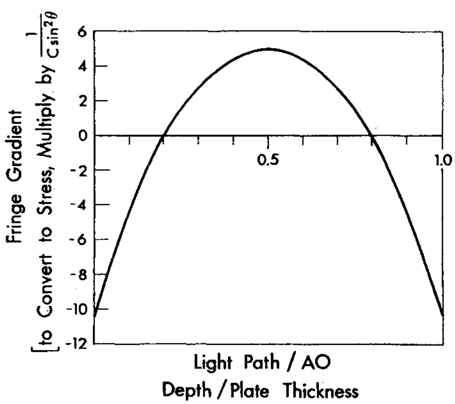


Fig. 6 Curve of fringe gradient and stress distribution across the thickness of the plate.

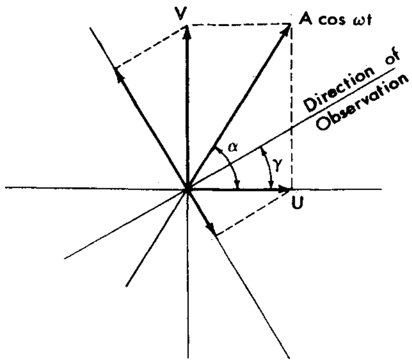


Fig. 7 Sketch showing the vibrational component normal to the direction of observation.

This expression would be equal to zero only if r is equal to zero, since r cannot reverse its sign across the thickness of the plate. Hence, it is concluded that at least r is negligible.

3) In addition to the preceding equilibrium check, another experiment was made with $\theta = 82°$. The fringe gradient was again calculated. A good agreement was found between the results of this experiment and that at $\theta = 84°$.

4) Results show that an approximately parabolic distribution of stress does exist across the thickness of a thermally tempered glass plate. However, the distribution in chemically tempered glass is not known. Experiments in chemically tempered glass are in preparation.

Appendix

It can be shown that the vibrational component normal to the direction of observation defined by γ, Fig. 7, is

$$A \cos\alpha \sin\gamma \cos\omega t - A \sin\alpha \cos\gamma \cos(\omega t + \phi) = A[(\cos\alpha \sin\gamma - \sin\alpha \cos\gamma \cos\phi)\cos\omega t + \sin\alpha \cos\gamma \sin\phi \sin\omega t] \quad \text{(A1)}$$

The square of the amplitude is

$$A^2[(\cos\alpha \sin\gamma - \sin\alpha \cos\gamma \cos\phi)^2 + (\sin\alpha \cos\gamma \sin\phi)^2] = A^2(\cos^2\alpha \sin^2\gamma + \sin^2\alpha \cos^2\gamma - \tfrac{1}{2} \sin 2\alpha \sin 2\gamma \cos\phi) \quad \text{(A2)}$$

Hence, the intensity of scattered light is

$$I = KA^2(\cos^2\alpha \sin^2\gamma + \sin^2\alpha \cos^2\gamma - \tfrac{1}{2} \sin 2\alpha \sin 2\gamma \cos\phi) \quad \text{(A3)}$$

References

[1] Haward, R. N., *The Strength of Plastics and Glass*, Cleaver-Hume Press, London, 1949.

[2] Hillig, H. S., *Symposium sur la Resistance Mecanique du Verre*, Florence, 1961.

[3] Krenzke, M., "Exploratory Tests of Long Glass Cylinders under External Hydrostatic Pressure," Rept. 1641, Aug. 1962, David Taylor Model Basin.

[4] Acloque, P. and Guillemet, C., "Method for the Photoelastic Measurement of Stresses in Equilibrium in the Thickness of a Plate," *Proceedings of the Stress Analysis Conference*, Institute of Physics, Delft, 1956, pp. 71–76.

[5] Bateson, S. et al., "Stress Measurements in Tempered Glass Plates by Scattered Light Method with a Laser Source," *American Ceramic Society Bulletin*, Vol. 45, No. 2, Feb. 1966, pp. 193–198.

[6] Cheng, Y. F., "A Scattered Light Photoelastic Method for the Determination of Tempered Stresses in Aircraft Windshields," *Strain*, Vol. 3, No. 2, April 1967, pp. 17–22.

[7] Coker, E. G. and Filon, L. N. G., *A Treatise on Photoelasticity*, Cambridge University Press, Cambridge, England, 1931.

[8] Condon, E. U. and Odishaw, H., eds., *Handbook of Physics*, McGraw-Hill, 1958, p. 3–85.

[9] Frocht, M. M., *Photoelasticity*, Vol. II, Wiley, New York, 1948.

[10] Weller, R., "Three-Dimensional Photoelasticity using Scattered Light," *Journal of Applied Physics*, Vol. 12, No. 8, Aug. 1941, pp. 610–616.